Dynamic model for the reliability problem in telecommunication network

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Abstract

In this work, we have defined the dynamic model for the telecommunication network problem. The network reliability measures have been estimated by means of two methods: the first is based on the lower and the upper bounds, when the second is based on discrete events simulation. We have given some numerical results on Arpanet network example.

1 Introduction

The network reliability evaluation is become very important in the dimensioning and the design of telecommunication network. The techniques of optimization of network depend on reliability measures, this leads to a necessity to conceive techniques to compute the network reliability.

Several models have been proposed recently for the computation of telecommunication network reliability. In the majority of these models (*Brecht (1985), Cancela and El Khadiri (1996))*, the network is defined by an undirected connected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consisting of a set of nodes $\mathcal{V}$ and a set of connecting links $\mathcal{E}$. Each link e_i can be operational with probability r_i or failed with probability $1 - r_i$. The reliability measure that is more studied in this context is the probability of success of communication between nodes of $\mathcal{V}$ noted by $R_{\mathcal{G}}$.

The measure $R_{\mathcal{G}}$ is a punctual measure and is independent of time. For necessities of design at long range, it will be interesting to evaluate the network reliability at function of time. In this paper, we propose a dynamic model of telecommunication network reliability evaluation by taking into account the temporal concepts of reliability. In this model we compute three measures of reliability: the reliability function ($R(t)$), the failure rate $\lambda(t)$ and the mean time to failure (MTTF).

For the evaluation of the network reliability measures, we have proposed two methods, the first is based on the lower and the upper bounds, when the second is based on simulation techniques.

2 Description of the dynamical model

Consider a network defined by its graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \ldots, n\}$ is the set of vertices, and $\mathcal{E} = \{e_1, \ldots, e_m\}$ is the set of edges. For each edge $e_i (i = 1, \ldots, m)$ of the graph $\mathcal{G}$, we have associated a random variable T_i designing the lifetime of the edge e_i, with cumulative function F_i and reliability function $R_i(t) = Pr[T_i > t]$.

Denote by $X_i(t)$ Defining the state of the edge $e_i (i = 1, \ldots, m)$ at time t.

$$X_j(t) = \begin{cases} 1 & \text{if } \ T_i > t, \\ 0 & \text{otherwise.} \end{cases}$$

Let $\phi_{\mathcal{G}}(t)$ been the state of the graph $\mathcal{G}$ defined by

$$\phi_{\mathcal{G}}(t) = \begin{cases} 1 & \text{if the graph } \ \mathcal{G} \ \text{ is } \ \text{connected at time } \ t \\ 0 & \text{otherwise.} \end{cases}$$

We associate to the graph $\mathcal{G}$ its lifetime T, the state of the graph $\mathcal{G}$ at time t can be expressed by

$$\phi_{\mathcal{G}}(t) = \begin{cases} 1 & \text{if } T > t, \\ 0 & \text{otherwise.} \end{cases}$$

Then $\phi_{\mathcal{G}}(t) = \Phi_{\mathcal{G}}(X_1(t), \ldots, X_m(t))$, where $\Phi_{\mathcal{G}}$ is the structure function of the graph $\mathcal{G}$. The reliability of the graph $\mathcal{G}$ is given by

$$R(t) = Pr(\phi_{\mathcal{G}}(t) = 1) = \Pr(T > t). \tag{1}$$

3 Evaluation of the reliability function of a network

We will determine $R(t)$ the reliability function of the graph $\mathcal{G}$. For this, we propose 2 methods. The first is based on the bounds of reliability function when the second is based on discrete events simulation.

3.1 Evaluation of the bounds

The estimation of $R(t)$ (the reliability function) by the previous approach is very tedious for graphs with large sizes. We propose then the lower and the upper bounds of $R(t)$.
Consider $\mathcal{A}_1 = (\mathcal{V}, \mathcal{E}_1), \ldots, \mathcal{A}_l = (\mathcal{V}, \mathcal{E}_l)$ and $\mathcal{C}_1, \ldots, \mathcal{C}_p$ respectively the spanning trees and the minimal cut sets of the graph $\mathcal{G}$ [9]. Then the lower and the upper bounds of $R(t)$ are given by the following relation (*Barlow and Proschan (1975)*)

$$\prod_{j=1}^{p} \left(1 - \prod_{e_i \in \mathcal{C}_j} (1 - R_i(t)) \right) \le R(t) \le 1 - \prod_{j=1}^{l} \left(1 - \prod_{e_i \in \mathcal{E}_j} R_i(t) \right). \tag{2}$$

3.2 Proposed approach: discrete events simulation

We propose now a technique based on discrete events simulation to evaluate the reliability measures of a network as a function of time. The principle is to generate a sample of random variables $Y_1, Y_2, \ldots, Y_N$ corresponding to failure moments of N graphs $\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_N$ identical to the graph $\mathcal{G}$.

3.2.1 Principle of the method

We generate a sample of N random vectors $T^{(i)} = (T_1^{(i)}, T_2^{(i)}, \ldots, T_m^{(i)})$, $(i = 1, \ldots, N)$, corresponding to the graphs $\mathcal{G}_1, \ldots, \mathcal{G}_N$, where $T_j^{(i)} (j = 1, \ldots, m)$ is the random variable which corresponds to the failure moment of the edge e_j of the graph $\mathcal{G}_i$. It is generated according to the law of T_j corresponding to the lifetime of the edge e_j of the graph $\mathcal{G}_i$. For each vector $T^{(i)}$, $(i = 1, \ldots, N)$, we order with croissant manner its elements $T_1^{(i)}, T_2^{(i)}, \ldots, T_m^{(i)}$, we obtain so an arranged random vector $T^{*(i)} = (T_1^{*(i)}, T_2^{*(i)}, \ldots, T_m^{*(i)})$, where the instants $T_1^{*(i)}, T_2^{*(i)}, \ldots, T_m^{*(i)}$ correspond to the edges $e_{(1)}, e_{(2)}, \ldots, e_{(m)}$ respectively.
We construct progressively the graphs $\mathcal{G}_i^{*(1)}, \ldots, \mathcal{G}_i^{*(k)}, (1 \le k \le m)$, by testing if the graph is connected, until obtaining a disconnected graph $\mathcal{G}_i^{*(k)}$. The graph $\mathcal{G}_i^{*(j)}, (1 \le j \le k)$ is constructed as follows:

$$\mathcal{G}_i^{*(j)} = \mathcal{G}_i^{*(j-1)} - e_{(j)}, \quad j = 1, \ldots, k \;\text{ and }\; \mathcal{G}_i^{*(0)} = \mathcal{G}_i \tag{3}$$

We obtain so a generated random variable $Y_i, (i = 1, \ldots, N)$:

$$Y_i = T_k^{*(i)} = Min\{T_j^{*(i)}, j = 1, \ldots, m / \;\; \mathcal{G}_i^{*(j)} \;\text{ is disconnected}\} \tag{4}$$

3.2.2 Estimation of the reliability function $R(t)$

From the generated sample $Y_1, Y_2, \ldots, Y_N$ corresponding to the instants of failure of the graphs $\mathcal{G}_1, \mathcal{G}_2, , \mathcal{G}_N$ identical to the graph $\mathcal{G}$, we can estimate the reliability function $R_N^*(t)$ of the graph $\mathcal{G}$ by constructing the empirical repartition function $F_N^* = 1 - R_N^*$ defined by

$$F_N^*(t) = \frac{1}{N} \sum_{i=1}^{N} 1_{\{Y_i \le t\}} = \frac{N(t)}{N}, \tag{5}$$

where $N(t)$ designs the number of graphs failed at date t. This empirical estimator F_N^* is unbiased and convergent.

3.2.3 Estimation of the failure rate $\lambda(t)$

For a lifetime T with density function f and reliability function R, the failure rate $\lambda(t)$ is defined by the relation:

$$\lambda(t) = \frac{f(t)}{R(t)}. \tag{6}$$

From the generated sample, the search for the estimator of $\lambda(t)$ consists of searching the estimator of the density f. The method that is usually applied is the Kernel convolution. We have chosen the normal estimator.

3.2.4 Estimation of the mean time to failure: $MTTF$

The mean lifetime of the graph $\mathcal{G}$ is estimated by the empirical mean of the variables $Y_1, Y_2, \ldots, Y_N$:

$$\widehat{MTTF} = \frac{1}{N}\sum_{i=1}^{N} Y_i.$$

This estimator is unbiased and consisting.

4 Application to the Arpanet network

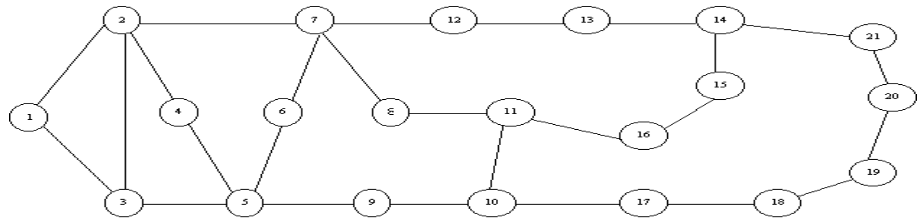

Fig. 1 Arpanet network topology

4.1 Presentation of numerical results

4.1.1 Reliability function $R(t)$

We have traced the curves of $R(t)$: the reliability function of the graph $\mathcal{G}$, for different values of λ in order to compare the different methods illustrated in this paper. The figures Fig. 2, Fig. 3 and Fig. 4 show a small difference between the obtained results. The discrete events simulation method gives an executing time equal to 0.40 second.

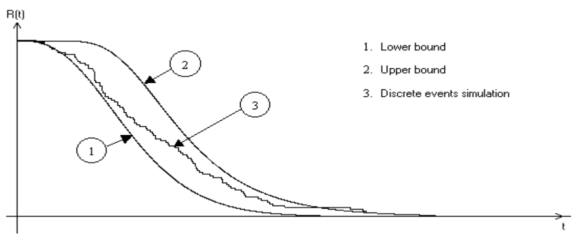

Fig. 2 Reliability function of the graph $\mathcal{G}$ for λ=0.5 failure/hour

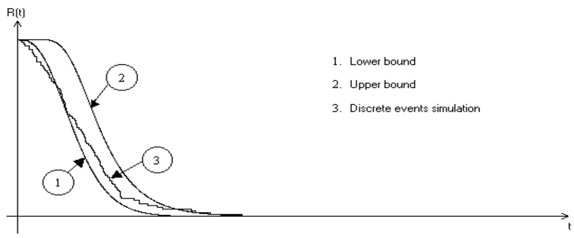

Fig. 3 Reliability function of the graph $\mathcal{G}$ for λ=1 failure/hour

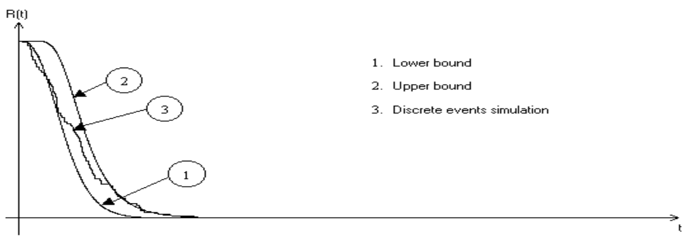

Fig. 4 Reliability function of the graph $\mathcal{G}$ for λ=1.5 failures/hour

4.1.2 Failure rate $\lambda(t)$

We have traced the curve of the failure rate $\lambda(t)$ of the graph $\mathcal{G}$ for different values of λ. The figures Fig. 5, Fig. 6 and Fig. 7 show that the function $\lambda(t)$ increases in t.

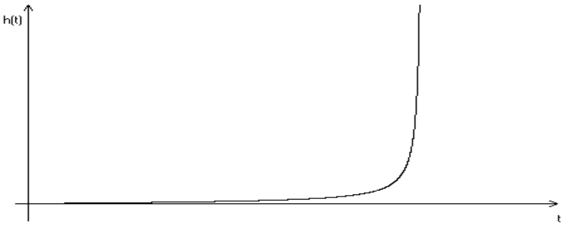

Fig. 5 Failure rate of the graph $\mathcal{G}$ for λ=0.5 failure/hour

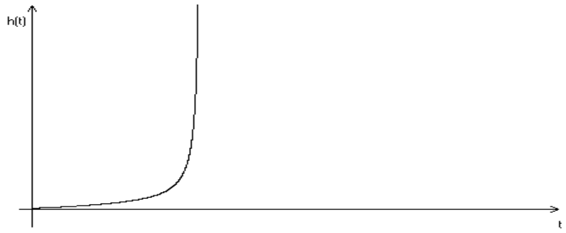

Fig.6 Failure rate of the graph $\mathcal{G}$ for λ=1 failures/hour

Fig. 7 Failure rate of the graph $\mathcal{G}$ for λ=1.5 failures/hour.

4.1.3 Mean time to failure: $MTTF$

The following table, gives values for $MTTF$: the mean time to failure of the graph $\mathcal{G}$ for different values of λ.

Table 1: mean time to failure of the graph $\mathcal{G}$.

λ	$MTTF$ (hour)
0.5	2.214
1	0.957
1.5	0.714

5 Conclusion

In this paper, we were interested to the evaluation of network reliability measures. We have defined a dynamical model by taking into account the temporal concepts of reliability theory. We have associated to each component of the network a random lifetime in order to evaluate the network reliability measures: the reliability function, the failure rate and the mean time to failure (MTTF). We have proposed two methods: the first is based on the lower and the upper bounds, when the second is based on simulation techniques. We have compared the results obtained on the Arpanet network example.

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